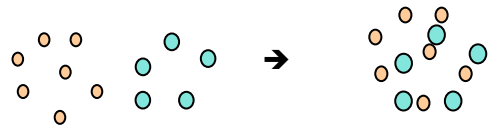
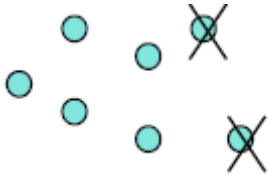
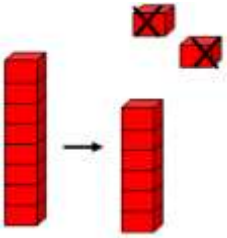
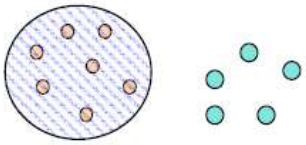
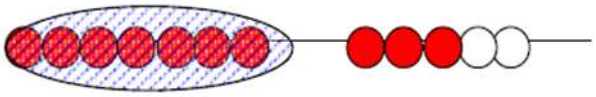
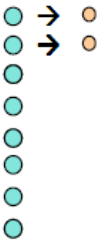
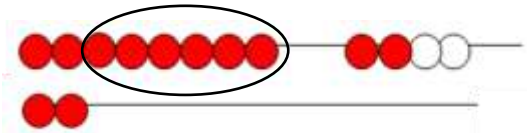
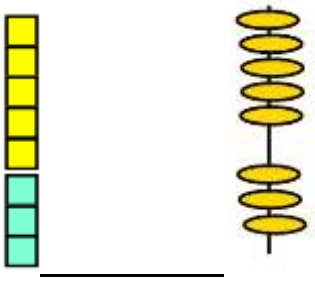
Addition	Subtraction
Combining two sets (aggregation) Putting together – two or more amounts or numbers are put together to make a total 7 + 5 = 12  Count one set, then the other set. Combine the sets and count again. Starting at 1. Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1. 	**Taking away (separation model)** Where one quantity is taken away from another to calculate what is left. 7 – 2 = 5  Multilink towers - to physically take away objects. 
Combining two sets (augmentation) Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number. Counters:  Start with 7, then count on 8, 9, 10, 11, 12 Bead strings:  Make a set of 7 and a set of 5. Then count on from 7.	**Finding the difference (comparison model)** Two quantities are compared to find the difference. 8 – 2 = 6 Counters:  Bead strings:  Make a set of 8 and a set of 2. Then count the gap.

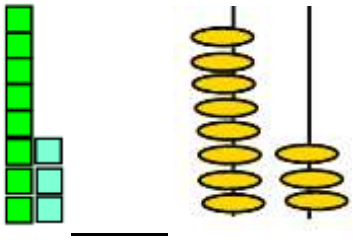
Explore practically using multilink, Numicon, lines and beads.



Count the total.

The calculation should be represented in a bar model or using rods.

Explore practically using multilink, Numicon, lines and beads.



Count the gap.

The calculation should be represented in a bar model or using rods.

Bridging through 10s

Compensation model (adding 9 and 11)

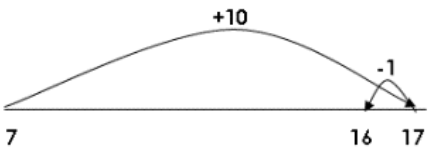
This stage is to aid mental fluency

7 + 9

Bead string:

Children find 7, then add on 10 and then adjust by removing 1.

Number line:

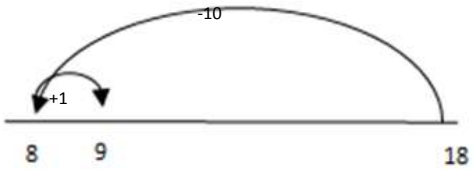


18 – 9

Bead string:

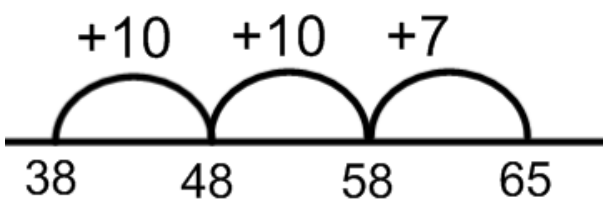
Children find 18, then subtract 10 and then adjust by adding 1.

Number line:



Working with larger numbers
Tens and ones + tens and ones

Numberline

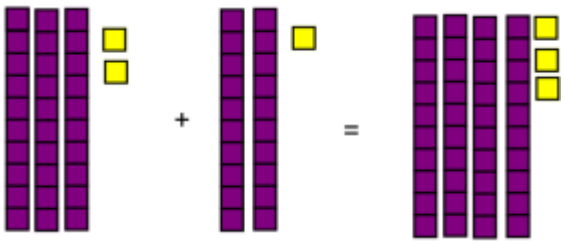


Add tens first, followed by ones.

Partitioning (Aggregation model)

32 + 21 = 53

Base 10 equipment:

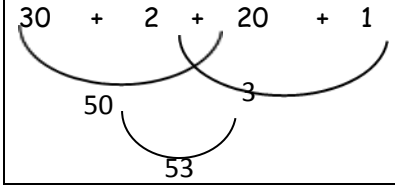


Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.

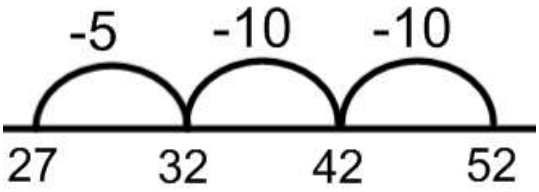
Begin counting with the ones in preparation for formal columnar method.

Show partitioning using number beads to support moving on to written calculations.

At this stage, children can begin to use an informal method to support, record and explain their method.



Numberline



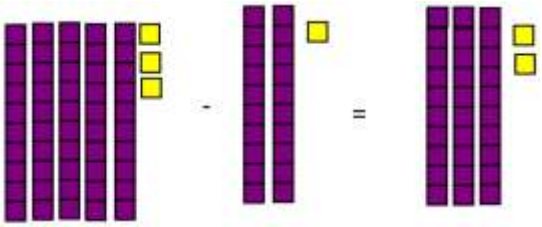
Subtract tens first, followed by ones.

Subtraction (Separation model)

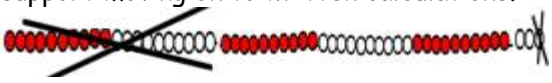
53 - 21 = 32

Base 10 equipment:

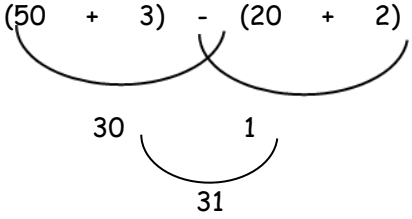
Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.



Show partitioning using number beads to support moving on to written calculations.



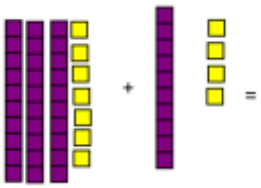
At this stage, children can began to use an informal method to support, record and explain their method.

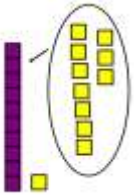


Bridging with larger numbers

Base 10 equipment:

37 + 14 = 51

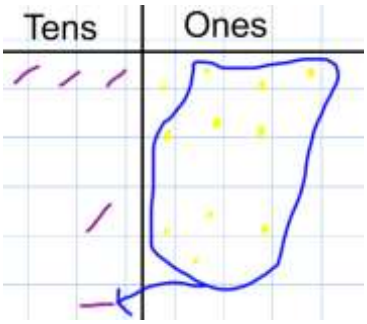




Exchange ones for tens.

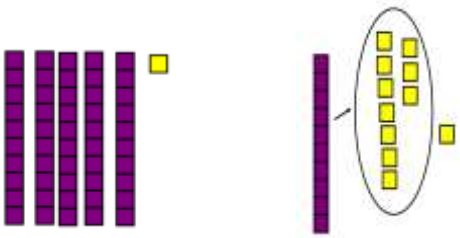
Count on from the biggest number.

Move onto pictorial:

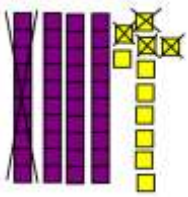


Base 10 equipment:

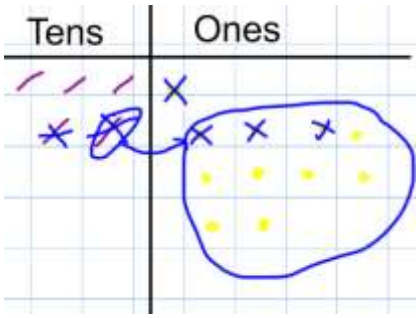
51 – 14 = 37



Exchange tens for ones.

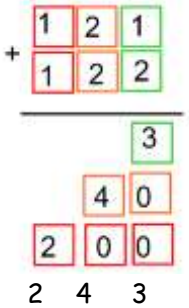


Move onto pictorial

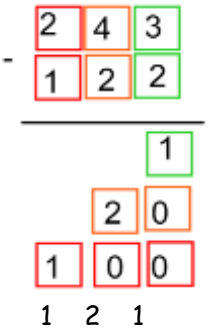


Expanded Vertical Method (optional)

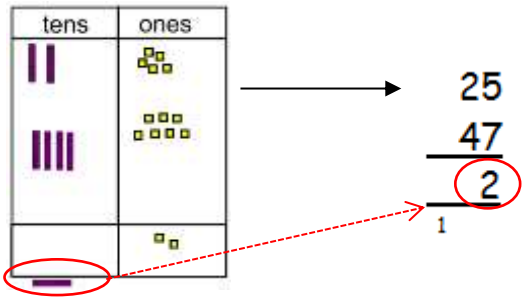
Expanded Method



Expanded Method

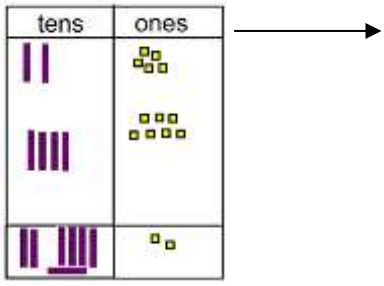


Compact method

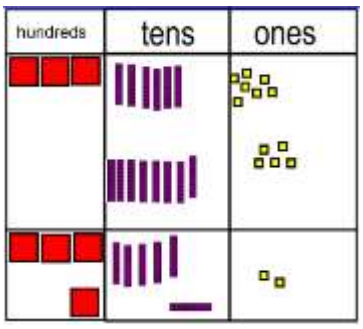


$$\begin{array}{r} 25 \\ 47 \\ \hline 2 \\ \hline 1 \end{array}$$

Leading to



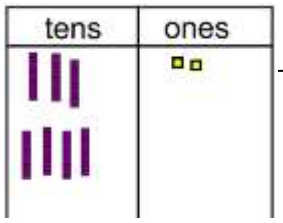
$$\begin{array}{r} 25 \\ 47 \\ \hline 72 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 367 \\ +\ 85 \\ \hline 452 \\ \hline 11 \end{array}$$

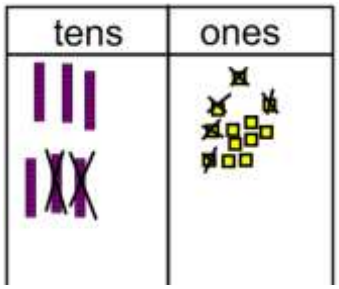
Children can also show this pictorially.

Compact decomposition



$$\begin{array}{r} 72 \\ -25 \\ \hline 47 \end{array}$$

$$\begin{array}{r} {}^{6}\not{7}\,{}^{1}2 \\ -25 \\ \hline \end{array}$$

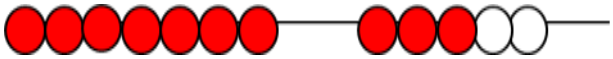


$$\begin{array}{r} {}^{6}\not{7}\,{}^{1}2 \\ -25 \\ \hline 47 \end{array}$$

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Base 10 equipment:

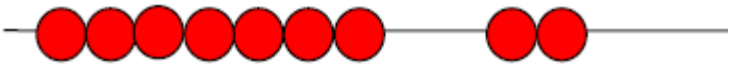
0.1 1.0 10.0

Addition of decimals

Aggregation model of addition

Counting both sets – starting at zero.

0.7 + 0.2 = 0.9

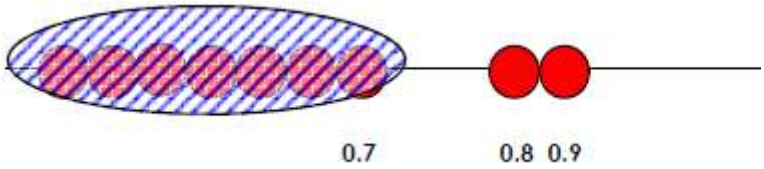


0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9



0.7 0.8 0.9

Bridging through 1.0

Encouraging connections with number bonds.

0.7 + 0.5 = 1.2

0.7 0.8 0.9 1.0 1.1 1.2

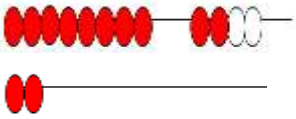
Subtraction of decimals

Take away model

0.9 – 0.2 = 0.7

Finding the difference (or comparison model):

0.8 – 0.2 =



Bridging through 1.0

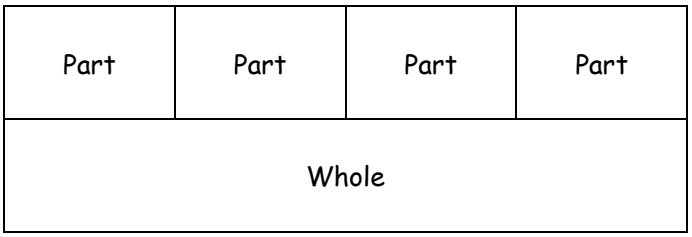
Encourage efficient partitioning.

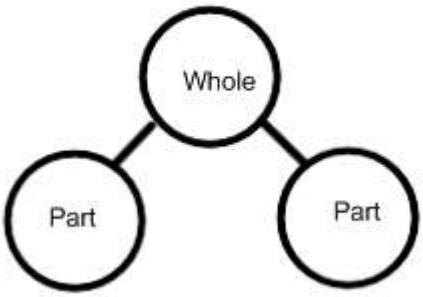
1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7

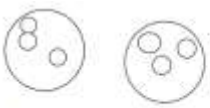
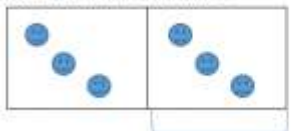
Gradation of difficulty- addition:	**Gradation of difficulty- subtraction:**
1. No exchange.	1. No exchange.
2. Breaking through ten.	2. Fewer digits in the answer.
3. Exchanging ones to tens.	3. Exchanging tens for ones.
4. Exchanging tens to hundreds.	4. Exchanging hundreds for tens
5. Exchanging ones to tens and tens to Hundreds.	5. Exchanging hundreds to tens and tens to ones.
6. More than two numbers in calculation, including use of exchange.	6. As 5 but with different number of digits.
7. As 6 but with different number of digits	7. Decimals up to 2 decimal places (same number of decimal places).
8. Decimals up to 2 decimal places (same number of decimal places)	8. Subtract two or more decimals with a range of decimal places.
9. Add two or more decimals with a range of decimal places.	

Multiplication and Division

Division should be taught as the inverse to multiplication. Pupils should see multiplication and division demonstrated in arrays, the bar model and the part-part-whole model as shown below:



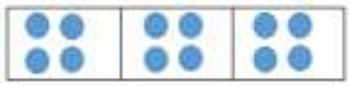


Multiplication	**Division**
Children should begin to write multiplication calculations such as 3 x 4 and recognise this as repeated addition, e.g. 4 + 4 + 4 **Repeated grouping/repeated addition** (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 	Children should begin by halving or sharing between two.  This can also be done in a bar so all 4 operations have a similar structure: 

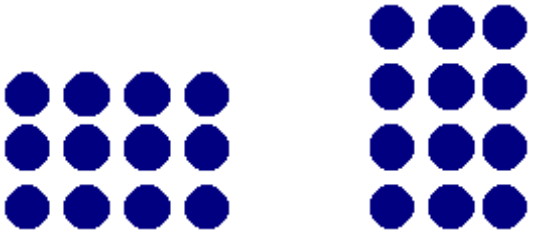
Children to represent the practical resources in a picture e.g.

XX XX XX
XX XX XX

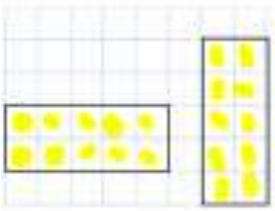
Use of a bar model for a more structured method



Use arrays to write a range of calculations, e.g. 3 x 4, 4 x 3…
These can be referred to as multiplication families.



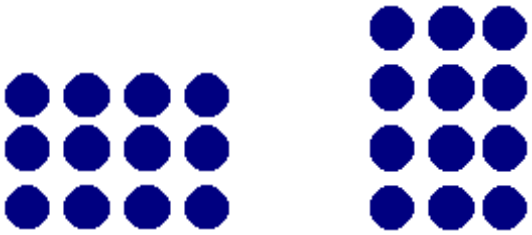
Children to draw the arrays



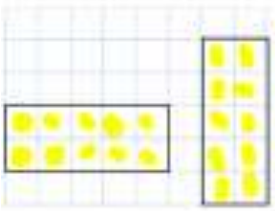
Understand division as repeated grouping and subtracting

6 ÷ 2

Use arrays to write a range of calculations, e.g.
12 divided by 4, 12 divided by 3…
These can be referred to as multiplication families.



Children to draw the arrays

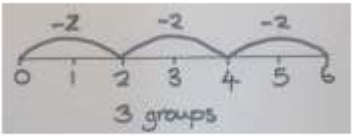


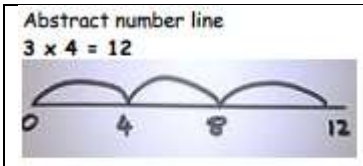
Use number lines to demonstrate calculations.

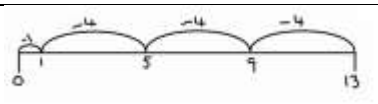
Use number lines to show repeated groups- 3 x 4

Use number lines and find remainders when using repeated subtraction.

Use of lollipop sticks to form wholes

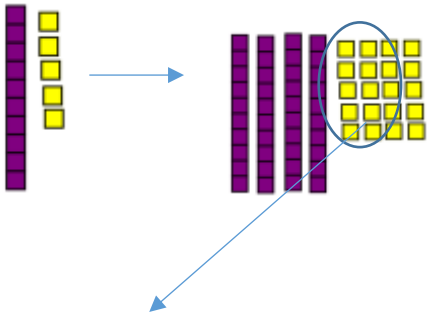


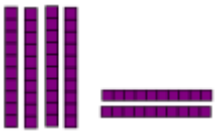




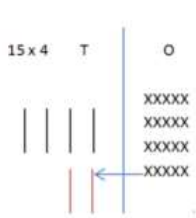
Partition to multiply

Begin with Base 10



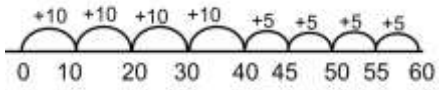


Move onto pictorial



or

Children can also use a numberline.

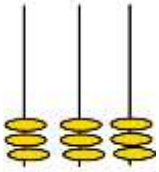


10 x 4 = 40
5 x 4 = 20

40 + 20 = 60

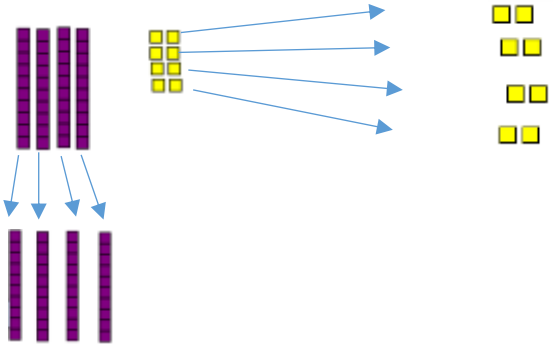
Sharing

Begin sharing in ones:

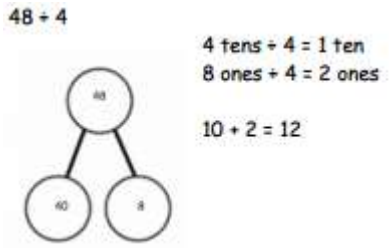


Use lines to do this.

Begin sharing tens and ones:



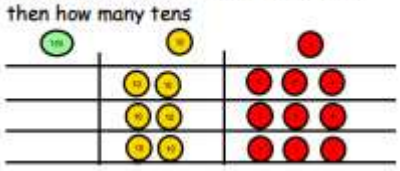
Move onto pictorial



Begin formal methods of multiplication

Begin formal methods of division

Make 23, 3 times. See how many ones, then how many tens



Moving onto pictorial

3 x 23

20 3

3 x 20 = 60

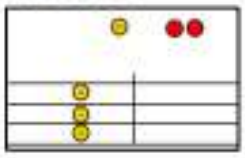
3 x 3 = 9

60 + 9= 69

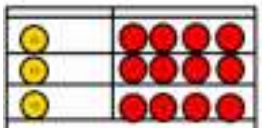
$$\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$$

Sharing using place value counters.

42 ÷ 3= 14

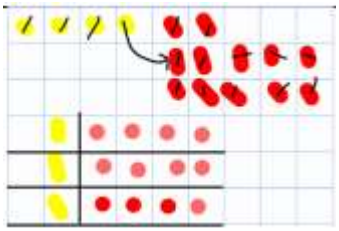


1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?

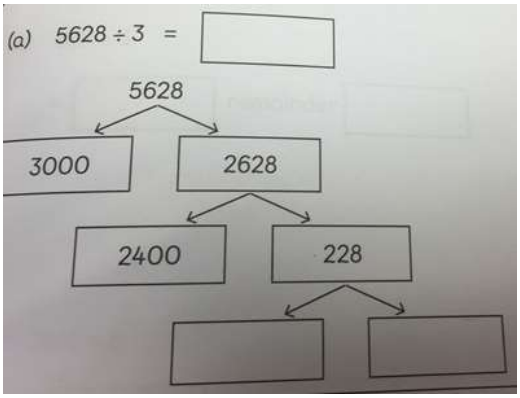


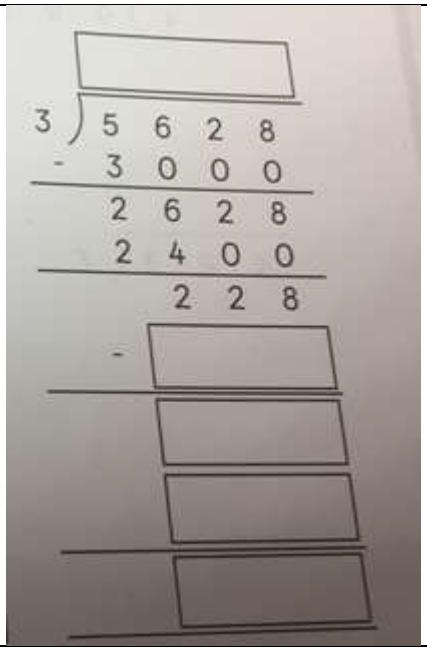
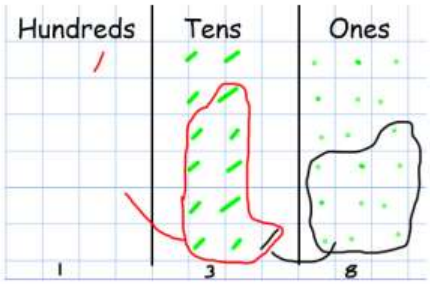
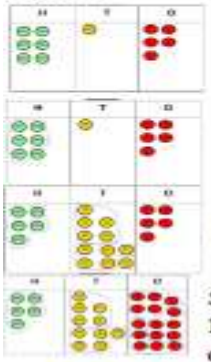
Exchange the ten for 10 ones and share out 12 ones

Move onto pictorial



Use vertical chunking to relate how many multiples fit into a number as laid out in MNP.



	
Move onto symbolic  6 x 23 = 23 x 6 138 1 1	**Move onto symbolic** **Use of the 'bus stop method'** using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- ***this can also be done using sharing!*** 615 ÷ 5  Step 1: make 615 Step 2: Circle your groups of 5 Step 3: Exchange 1H for 10T and circle groups of 5 Step 4: exchange 1T for 10ones and circles groups of 5 123 5 ⟌ 6¹1¹5
Long Multiplication At this point, chn should be confident with the abstract in order to move on. 3d x 3d; 4d x 2d	**Long Division**  Use the counters to demonstrate each stage of the division calculation so that children fully understand the exchange.

1 2 4 × 2 6 7 4 4 1 2 2 4 8 0 3 2 2 4 1 1 Answer: 3224	0 12 | 2544 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. 02 12 | 2544 24 1 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. 021 12 | 2544 24 14 12 2 Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. 0212 12 | 2544 24 14 12 24 24 0 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.